Physics 4A
Chapters 11: Impulse and Momentum

GENERAL PRINCIPLES

Law of Conservation of Momentum

The total momentum $\vec{P} = \vec{p}_1 + \vec{p}_2 + \cdots$ of an isolated system is a constant. Thus

$$\vec{P}_f = \vec{P}_i$$

Newton's Second Law

In terms of momentum, Newton's second law is

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Solving Momentum Conservation Problems

MODEL Choose an isolated system or a system that is isolated during at least part of the problem.

VISUALIZE Draw a pictorial representation of the system before and after the interaction.

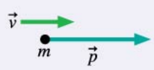
SOLVE Write the law of conservation of momentum in terms of vector components:

$$(p_{fx})_1 + (p_{fx})_2 + \cdots = (p_{ix})_1 + (p_{ix})_2 + \cdots$$
$$(p_{fy})_1 + (p_{fy})_2 + \cdots = (p_{iy})_1 + (p_{iy})_2 + \cdots$$

ASSESS Is the result reasonable?

IMPORTANT CONCEPTS

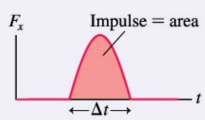
Momentum $\vec{p} = m\vec{v}$



Impulse $J_x = \int_{t_i}^{t_f} F_x(t)\, dt =$ area under force curve

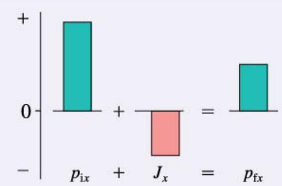
Impulse and momentum are related by the **momentum principle**

$$\Delta p_x = J_x$$



The impulse delivered to an object causes the object's momentum to change. This is an alternative statement of Newton's second law.

Momentum bar charts display the momentum principle $p_{fx} = p_{ix} + J_x$ in graphical form.

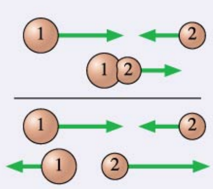


System A group of interacting particles.

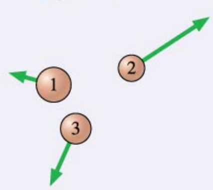
Isolated system A system on which there are no external forces or the net external force is zero.

APPLICATIONS

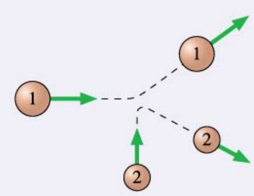
Collisions In a **perfectly inelastic collision,** two objects stick together and move with a common final velocity. In a **perfectly elastic collision,** they bounce apart and conserve mechanical energy as well as momentum.



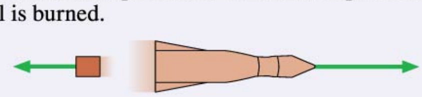
Explosions Two or more objects fly apart from each other. Their total momentum is conserved.



Two dimensions The same ideas apply in two dimensions. Both the x- and y-components of $\vec{P}$ must be conserved. This gives two simultaneous equations to solve.



Rockets The momentum of the exhaust-gas + rocket system is conserved. **Thrust** is the product of the exhaust speed and the rate at which fuel is burned.



Question and Example Problems from Chapter 11

Conceptual Question 11-4

A 0.2 kg plastic cart and a 20 kg lead cart can both roll without friction on a horizontal surface. Equal forces are used to push both carts forward for a time of 1 s, starting from rest. After the force is removed at t = 1 s, is the momentum of the plastic cart greater than, less than, or equal to the momentum of the lead cart? Explain.

11.4. When the question talks about forces, times, and momenta, we immediately think of the impulse-momentum theorem, which tells us that, to change the momentum of an object, we must exert a net external force on it over a time interval: $\Delta\vec{p} = \vec{F}_{\text{avg}}\Delta t$. Because equal forces are exerted over equal times, the impulses are equal and the changes in momentum are equal. Because both carts start from rest, the change in momentum of each is the same as the final momentum of each, so their final momenta are equal. Notice that, to answer the question, we do not need to know the mass of either cart, or even the specific time interval (as long as it is the same for both carts).

Conceptual Question 11-5

A 0.2 kg plastic cart and a 20 kg lead cart can both roll without friction on a horizontal surface. Equal forces are used to push both carts forward for a distance of 1 m, starting from rest. After traveling 1 m, is the momentum of the plastic cart greater than, less than, or equal to the momentum of the lead cart? Explain.

11.5. The impulse-momentum theory tells us that the change in momentum of an object is related to the net force on the object and the length of time the force was applied. Mathematically, $\Delta\vec{p} = \vec{F}_{\text{avg}}\Delta t$. The same force applied to the two carts results in a larger acceleration for the less massive plastic cart (Newton's second law), enabling it to travel the 1-m distance in a shorter time. Therefore, the plastic cart has a smaller change in momentum than the lead cart. Because the final momentum of each cart is equal to their change in momentum (zero initial momentum), the final momentum of the plastic cart is less than that of the lead cart.

Conceptual Question 11-8

Automobiles are designed with "crumple zones" intended to collapse in a collision. Use the ideas of this chapter to explain why.

11.8. The impulse-momentum theory states that a change in an object's momentum results when a net force is applied to the object for some time interval; $\Delta\vec{p} = \vec{F}_{\text{avg}}\Delta t$. Stopping an automobile requires changing its momentum from some to none. This change can be accomplished with a small force over a long time interval or a large force over a short time interval. The crumple zone that collapses during an automobile collision lengthens the time interval during which the automobile is stopped, resulting in a smaller force on the passengers as they also come to a stop.

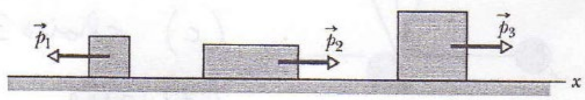
Conceptual Question 11-9

A golf club continues forward after hitting the golf ball. Is momentum conserved in the collision? Explain, making sure you are careful to identify "the system".

11.9. The club and ball form a system. The interaction force when the club and ball collide is very large compared to other forces at the time of collision, such as gravity and the force of the golfer on the club. So, in this impulse approximation, momentum is conserved during the collision. After the club hits the ball, it will give the ball some of its momentum. The club can continue to move forward as long as the momentum the ball obtains is less than the initial momentum of the club. Note that the momentum conservation is valid only if we consider the short time between just before and just after the collision. The wider we make the time window, the more time gravity and the golfer have to influence the motion of the club and ball, so that momentum conservation would no longer hold for the club-ball system.

Conceptual Question 11-A
A container sliding along an x axis on a frictionless surface explodes into three pieces. The pieces then move along the x axis in the directions indicated in the figure below. The following table gives four sets of magnitudes (in kg·m/s) for the momenta 1, 2, and 3 of the pieces. Rank the sets according to the initial speed of the container, greatest first.

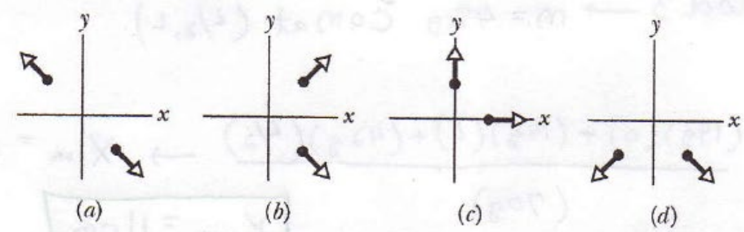
	p_1	p_2	p_3		p_1	p_2	p_3
(a)	10	2	6	(b)	10	6	2
(c)	2	10	6	(d)	6	2	10



⇒ can rank by $|p_3 + p_2 - p_1|$

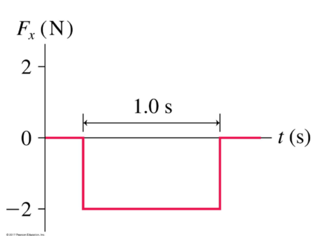
c, d, a=b

Conceptual Question 11-B
In the four situations indicated in the figure below, an object explodes into two equal-mass fragments when the object is at the origin of the coordinate system. The velocity vectors of the fragments are indicated; they are directed either along an axis or at 45° to an axis. For each situation determine the direction of travel of the object before the explosion, or note that it was stationary.



a) stationary
b) to the right
c) at 45°
d) straight downward

Problem 11-8
A 2.0 kg object is moving to the right with a speed of 1.0 m/s when it experiences the force shown in the figure below. What are the object's speed and direction after the force ends?



11.8. Model: Model the object as a particle and the interaction with the force as a collision.
Solve: Using the equations

$$p_{fx} = p_{ix} + J_x \text{ and } J_x = \int_{t_i}^{t_f} F_x(t)dt = \text{area under force curve}$$

$$(2.0\text{ kg})v_{fx} = (2.0\text{ kg})(1.0\text{ m/s}) + (\text{area under the force curve})$$

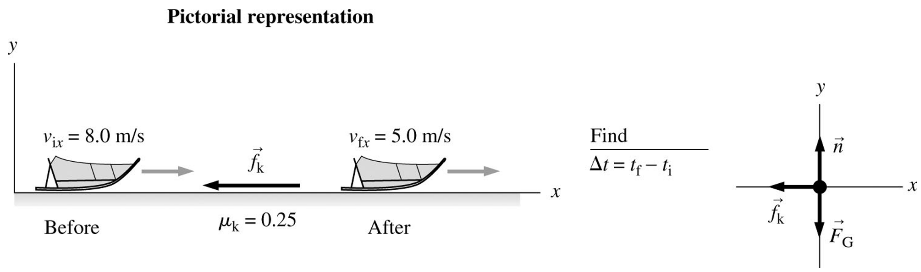
$$v_{fx} = (1.0\text{ m/s}) + \frac{1}{2.0\text{ kg}}(1.0\text{ s})(-2.0\text{ N}) = 0.0\text{ m/s}$$

Assess: For an object with positive velocity, a positive impulse increases the object's speed. The opposite is true for an object with negative velocity.

Problem 11-10

A sled slides along a horizontal surface on which the coefficient of kinetic friction is 0.25. Its velocity at point A is 8.0 m/s and at point B is 5.0 m/s. Use the momentum principle to find how long the sled takes to travel from A to B.

11.10. Model: Use the particle model for the sled, the model of kinetic friction, and the impulse-momentum theorem.
Visualize:



Note that the force of kinetic friction f_k imparts a negative impulse to the sled.
Solve: Using $\Delta p_x = J_x$, we have

$$p_{fx} - p_{ix} = \int_{t_i}^{t_f} F_x(t)dt = -f_k\int_{t_i}^{t_f} dt = -f_k\Delta t \quad\Rightarrow\quad mv_{fx} - mv_{ix} = -\mu_k n\Delta t = -\mu_k mg\Delta t$$

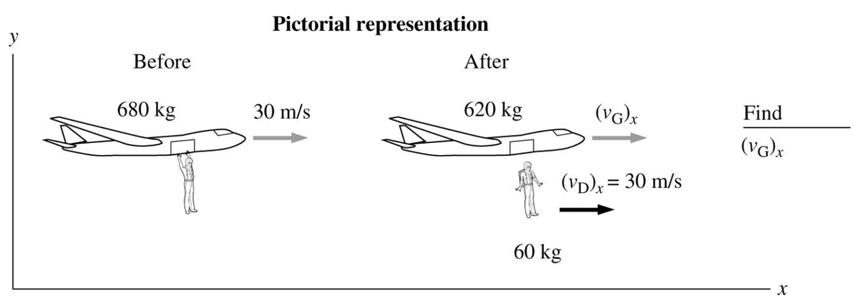
We have used the model of kinetic friction $f_k = \mu_k n$, where μ_k is the coefficient of kinetic friction and n is the normal (contact) force by the surface. The force of kinetic friction is independent of time and was therefore taken out of the impulse integral. Thus,

$$\Delta t = \frac{1}{\mu_k g}(v_{ix} - v_{fx}) = \frac{(8.0\text{ m/s} - 5.0\text{ m/s})}{(0.25)(9.8\text{ m/s}^2)} = 1.2\text{ s}$$

Problem 11-16

A 10-m-long glider with a mass 680 kg (including the passengers) is gliding horizontally through the air at 30 m/s when a 60 kg skydiver drops out by releasing his grip on the glider. What is the gliders's velocity just after the skydiver lets go?

11.16. Model: Choose skydiver + glider to be the system in the impulse approximation. Ignore air resistance.
Visualize:



Note that there are no *external* forces in the x-direction (ignoring friction in the impulse approximation), implying conservation of momentum along the x-direction.
Solve: The momentum conservation equation $p_{fx} = p_{ix}$ gives

$$(680\text{ kg} - 60\text{ kg})(v_G)_x + (60\text{ kg})(v_D)_x = (680\text{ kg})(30\text{ m/s})$$

Immediately after release, the skydiver's horizontal velocity is still $(v_D)_x = 30\text{ m/s}$ because he experiences no net horizontal force. Thus

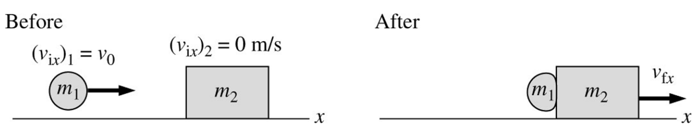
$$(620\text{ kg})(v_G)_x + (60\text{ kg})(30\text{ m/s}) = (680\text{ kg})(30\text{ m/s}) \quad \Rightarrow \quad (v_G)_x = 30\text{ m/s}$$

Assess: The skydiver's motion in the vertical direction has *no* influence on the glider's horizontal motion. Notice that we did not need to invoke conservation of momentum to solve this problem. Because there are no external horizontal forces acting on either the skydiver or the glider, neither will change their horizontal speed when the skydiver lets go!

Problem 11.24

A 50-g ball of clay traveling at speed v_0 hits and sticks to a 1.0 kg brick sitting at rest on a frictionless surface. **(a)** What is the speed of the brick after the collision (as a function of v_0)? **(b)** What percentage of the mechanical energy is lost in this collision?

11.24. Model: This is the case of a perfectly inelastic collision. Momentum is conserved because no external force acts on the system (clay + brick). We also represent our system as a particle.
Visualize:



Solve: (a) The conservation of momentum equation $p_{fx} = p_{ix}$ is

$$(m_1 + m_2)v_{fx} = m_1(v_{ix})_1 + m_2(v_{ix})_2$$

Using $(v_{ix})_1 = v_0$ and $(v_{ix})_2 = 0,$ we get

$$v_{fx} = \frac{m_1}{m_1 + m_2}(v_{ix})_1 = \frac{0.050\text{ kg}}{(1.0\text{ kg} + 0.050\text{ kg})}(v_{ix})_1 = 0.0476(v_{ix})_1 = 0.0476\, v_0$$

The brick is moving with speed $0.048v_0$.
(b) The initial and final kinetic energies are given by

$$K_i = \frac{1}{2}m_1(v_{ix})_1^2 + \frac{1}{2}m_2(v_{ix})_2^2 = \frac{1}{2}(0.050\text{ kg})v_0^2 + \frac{1}{2}(1.0\text{ kg})(0\text{ m/s})^2 = (0.025\text{ kg})v_0^2$$

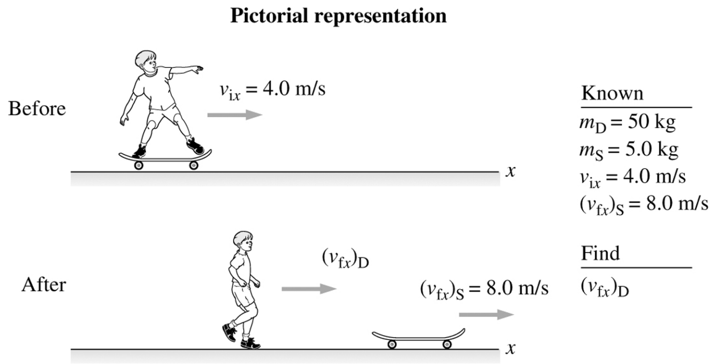
$$K_f = \frac{1}{2}(m_1 + m_2)v_{fx}^2 = \frac{1}{2}(1.0\text{ kg} + 0.050\text{ kg})(0.0476)^2 v_0^2 = 0.00119\, v_0^2$$

The percent of energy lost $= \left(\frac{K_i - K_f}{K_i}\right) \times 100\% = \left(1 - \frac{0.00119}{0.025}\right) \times 100\% = 95\%$

Problem 11-28

Dan is gliding on his skateboard at 4.0 m/s. He suddenly jumps backward off the skateboard, kicking the skateboard forward at 8.0 m/s. How fast is Dan going as his feet hit the ground? Dan's mass is 50 kg and the skateboard's mass is 5.0 kg.

11.28. Model: We will define our system to be Dan + skateboard, and their interaction as an explosion. While friction is present between the skateboard and the ground, it is negligible in the impulse approximation.
Visualize:



The system has nonzero initial momentum p_{ix}. As Dan (D) jumps backward off the gliding skateboard (S), the skateboard will move forward so that the final total momentum of the system p_{fx} is equal to p_{ix}.

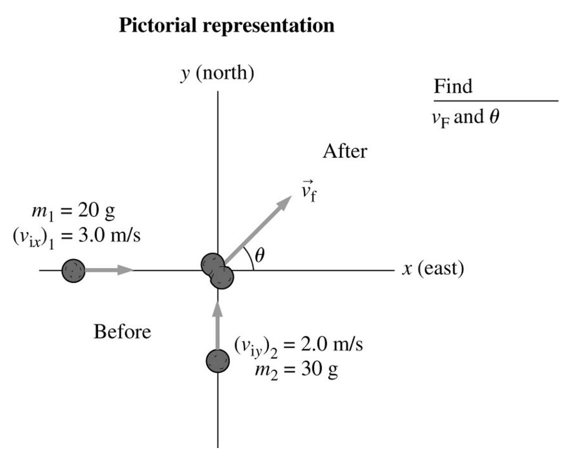
Solve: We have $m_S(v_{fx})_S + m_D(v_{fx})_D = (m_S + m_D)v_{ix}$. Thus,

$$(5.0\ \text{kg})(8.0\ \text{m/s}) + (50\ \text{kg})(v_{fx})_D = (5.0\ \text{kg} + 50\ \text{kg})(4.0\ \text{m/s}) \quad \Rightarrow \quad (v_{fx})_D = 3.6\ \text{m/s}$$

Problem 11-33

A 20 g ball of clay traveling east at 3.0 m/s collides with a 30 g ball of clay traveling north at 2.0 m/s. What are the speed and the direction of the resulting 50 g ball of clay? Give your answer as an angle north of east.

11.33. Model: This problem deals with the conservation of momentum in two dimensions in an inelastic collision.
Visualize:



Solve: The conservation of momentum equation $\vec{p}_{\text{before}} = \vec{p}_{\text{after}}$ gives

$$m_1(v_{ix})_1 + m_2(v_{ix})_2 = (m_1 + m_2)v_{fx},\ m_1(v_{iy})_1 + m_2(v_{iy})_2 = (m_1 + m_2)v_{fy}$$

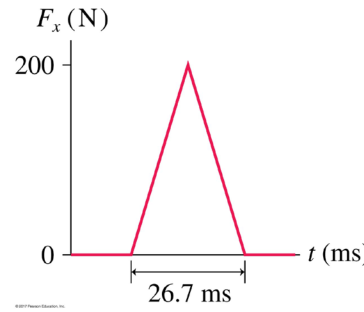
Substituting in the given values, we find

$$(0.020\ \text{kg})(3.0\ \text{m/s}) + 0.0\ \text{kg m/s} = (0.020\ \text{kg} + 0.030\ \text{kg})v_f \cos\theta$$
$$0.0\ \text{kg m/s} + (0.030\ \text{kg})(2.0\ \text{m/s}) = (0.020\ \text{kg} + 0.030\ \text{kg})v_f \sin\theta$$
$$v_f \cos\theta = 1.2\ \text{m/s},\ v_f \sin\theta = 1.2\ \text{m/s}$$
$$v_f = \sqrt{(1.2\ \text{m/s})^2 + (1.2\ \text{m/s})^2} = 1.7\ \text{m/s},\ \theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}(1) = 45°$$

The ball of clay moves 45° north of east at 1.7 m/s.

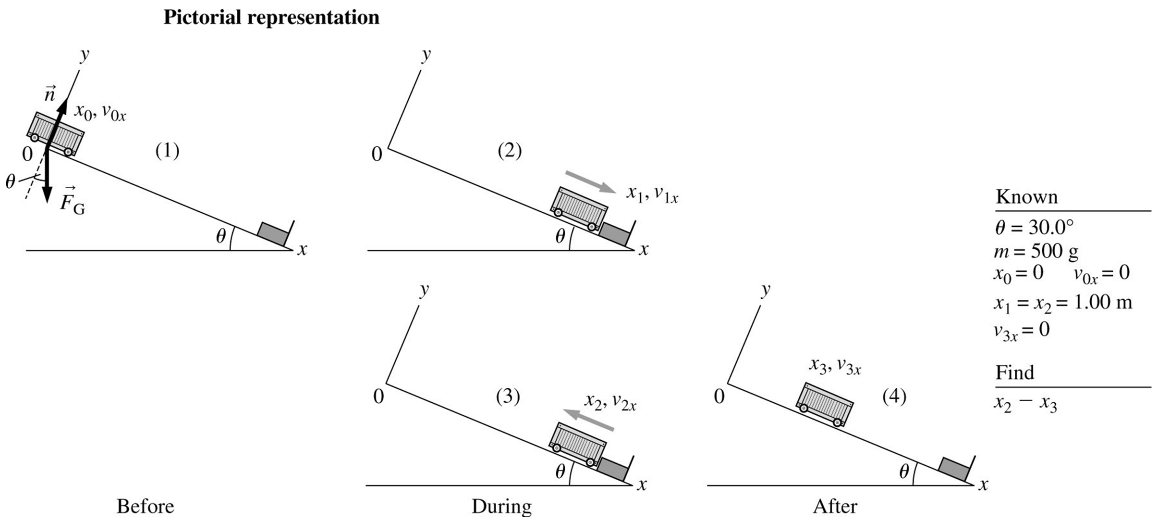
Problem 11-40

A 500 g cart is released from rest 1.0 m from the bottom of a frictionless, 30.0° ramp. The cart rolls down the ramp and bounces off a rubber block at the bottom. The figure shows the force during the collision. After the cart bounces, how far does it roll back up the ramp?



11.40. Model: Model the cart as a particle sliding down a frictionless ramp. The cart is subjected to an impulsive force when it comes in contact with a rubber block at the bottom of the ramp. We shall use the impulse-momentum theorem and the constant-acceleration kinematic equations.

Visualize:



Solve: From the free-body diagram on the cart, Newton's second law applied to the system before the collision gives

$$\Sigma(F)_x = F_G \sin\theta = ma_x \quad \Rightarrow \quad a_x = \frac{mg\sin\theta}{m} = g\sin 30.0° = \frac{9.81 \text{ m/s}^2}{2} = 4.905 \text{ m/s}^2$$

Using this acceleration, we can find the cart's speed just before its contact with the rubber block:

$$v_{1x}^2 = v_{0x}^2 + 2a_x(x_1 - x_0) = 0 \text{ m}^2/\text{s}^2 + 2(4.905 \text{ m/s}^2)(1.00 \text{ m} - 0 \text{ m}) \quad \Rightarrow \quad v_{1x} = 3.132 \text{ m/s}$$

Now we can use the impulse-momentum theorem to obtain the velocity just after the collision:

$$mv_{2x} = mv_{1x} + \int F_x dt = mv_{1x} + \text{area under the force graph}$$

$$(0.500 \text{ kg})v_{2x} = (0.500 \text{ kg})(3.13 \text{ m/s}) - \tfrac{1}{2}(200 \text{ N})(26.7 \times 10^{-3} \text{ s}) \quad \Rightarrow \quad v_{2x} = -2.208 \text{ m/s}$$

Note that the given force graph is positive, but in this coordinate system the impulse of the force is to the left (i.e., up the slope). That is the reason to put a minus sign while evaluating the $\int F_x dt$ integral.

We can once again use a kinematic equation to find how far the cart will roll back up the ramp:

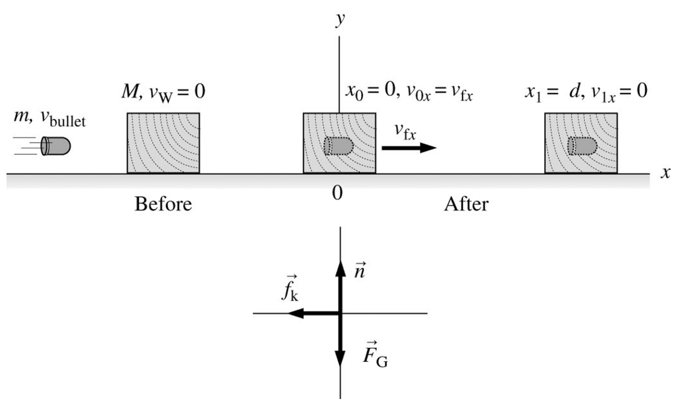
$$v_{3x}^2 = v_{2x}^2 + 2a_x(x_3 - x_2) \quad \Rightarrow \quad (0 \text{ m/s})^2 = (-2.208 \text{ m/s})^2 + 2(-4.905 \text{ m/s}^2)(x_3 - x_2) \quad \Rightarrow \quad (x_3 - x_2) = 0.497 \text{ m}$$

Problem 11-49

(a) A bullet of mass m is fired into a block of mass M that is at rest. The block, with the bullet embedded, slides distance d across a horizontal surface. The coefficient of kinetic friction is μ_k. Find a mathematical expression for the bullet's speed v_{bullet}. **(b)** What is the speed of a 10-g bullet that, when fired into a 10-kg stationary wood block, causes the block to slide 5.0 cm across a wood table ($\mu_k = 0.20$)?

11.49. Model: Model the bullet and block as particles. This is an isolated system because any frictional force during the brief collision period is going to be insignificant. Within the impulse approximation, the momentum of our system will be conserved in the collision. After the collision, we will consider the frictional force and apply Newton's second law and kinematic equations to find the distance traveled by the block + bullet.

Visualize:



Solve: **(a)** Applying conservation of momentum to the collision gives

$$mv_{bullet} + Mv_W = (m+M)v_{fx} \Rightarrow v_{bullet} = \frac{m+M}{m}v_{fx}$$

The speed v_{ex} can be found from the kinematics equation

$$v_{1x}^2 = v_{0x}^2 + 2ad = v_{fx}^2 + 2ad \Rightarrow v_{fx} = \sqrt{-2ad}$$

The acceleration in the x-direction may be found using Newton's second law and the friction model. Because the block does not accelerate in the y-direction, the normal force must be the same magnitude as the force due to gravity (Newton's second law). Thus, the frictional force is $f_k = -\mu_k n = -\mu_k(m+M)g$, where the negative sign indicates that the force acts in the negative x-direction. Newton's second law then gives the acceleration of the block as

$$a = F_{net}/(m+M) = -\mu_k(m+M)g/(m+M) = -\mu_k g$$

Inserting this into the expression for v_{ex} gives

$$v_{fx} = \sqrt{-2ad} = \sqrt{2\mu_k gd}$$

Finally, we insert this expression for v_{ex} into the expression for the bullet's velocity to find

$$v_{bullet} = \frac{m+M}{m}\sqrt{2\mu_k gd}$$

(b) Inserting the given quantities gives

$$v_{bullet} = \frac{0.010 \text{ kg} + 10 \text{ kg}}{0.010 \text{ kg}}\sqrt{2(0.20)(9.8 \text{ m/s}^2)(0.050 \text{ m})} = 4.4 \times 10^2 \text{ m/s}$$

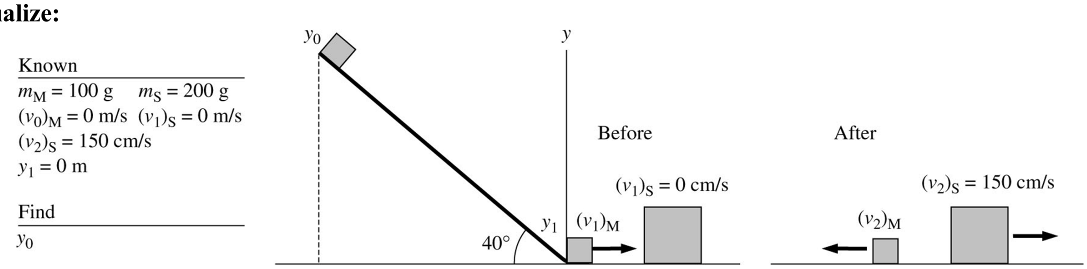
Assess: If we let the bullet's mass go to zero, we see that the bullet's speed goes to infinity, which is reasonable because a zero-mass bullet would need an infinite speed to make the block move. If the bullet's mass goes to infinity, the bullet's speed would go to $\sqrt{2\mu_k gd}$, which is just the result for the initial speed of an object that decelerates to a stop at a constant rate $(\mu_k g)$ over a distance d. In other words, the block becomes insignificant compared to the infinite-mass bullet.

Problem 11-55

A 100 g granite cube slides down a 40° frictionless ramp. At the bottom, just as it exits onto a horizontal table, it collides with a 200 g steel cube at rest. How high above the table should the granite cube be released to give the steel cube a speed of 150 cm/s.

11.55. Model: Model the marble and the steel ball as particles. We will assume an elastic collision between the marble and the ball, and apply the conservation of momentum and the conservation of energy equations. We will also assume zero rolling friction between the marble and the incline.

Visualize:



This is a two-part problem. In the first part, we will apply the conservation of energy equation to find the marble's speed as it exits onto a horizontal surface. We have put the origin of our coordinate system on the horizontal surface just where the marble exits the incline. In the second part, we will consider the elastic collision between the marble and the steel ball.

Solve: The conservation of energy equation $K_1 + U_{g1} = K_0 + U_{g0}$ gives us:

$$\frac{1}{2}m_M(v_1)_M^2 + m_M g y_1 = \frac{1}{2}m_M(v_0)_M^2 + m_M g y_0$$

Using $(v_0)_M = 0$ m/s and $y_1 = 0$ m, we get $\frac{1}{2}(v_1)_M^2 = gy_0 \Rightarrow (v_1)_M = \sqrt{2gy_0}$. When the marble collides with the steel ball, the elastic collision gives the ball velocity

$$(v_2)_S = \frac{2m_M}{m_M + m_S}(v_1)_M = \frac{2m_M}{m_M + m_S}\sqrt{2gy_0}$$

Solving for y_0 gives

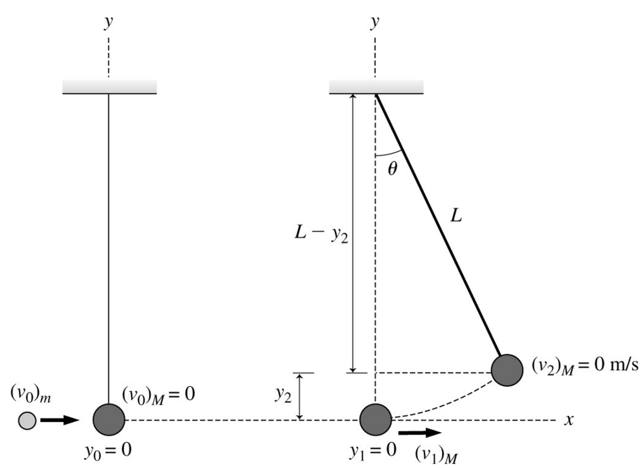
$$y_0 = \frac{1}{2g}\left[\frac{m_M + m_S}{2m_M}(v_2)_S\right]^2 = 0.258\text{ m} = 25.8\text{ cm}$$

Problem 11.63

A 20-g ball is fired horizontally with speed v_0 toward a 100-g ball hanging motionless from a 1.0-m-long string. The balls undergo a head-on, perfectly elastic collision, after which the 100 g ball swings out to a maximum angle $\theta_{max} = 50^\circ$. What was v_0?

11.63. Model: We can divide this problem into two parts. First, we have an elastic collision between the 20 g ball (m) and the 100 g ball (M). Second, the 100 g ball swings up as a pendulum.

Visualize:



The figure shows three distinct moments of time: the time before the collision, the time after the collision but before the two balls move, and the time the 100 g ball reaches its highest point. We place the origin of our coordinate system on the 100 g ball when it is hanging motionless.

Solve: For a perfectly elastic collision, the ball moves forward with speed

$$(v_1)_M = \frac{2m_m}{m_m + m_M}(v_0)_m = \frac{1}{3}(v_0)_m$$

In the second part, the sum of the kinetic and gravitational potential energy is conserved as the 100 g ball swings up after the collision. That is, $K_2 + U_{g2} = K_1 + U_{g1}$. We have

$$\frac{1}{2}M(v_2)_M^2 + Mgy_2 = \frac{1}{2}M(v_1)_M^2 + Mgy_1$$

Using $(v_2)_M = 0$ J, $(v_1)_M = \frac{(v_0)_m}{3}$, $y_1 = 0$ m, and $y_2 = L - L\cos\theta$, the energy equation simplifies to

$$g(L - L\cos\theta) = \frac{1}{2}\frac{(v_0)_m^2}{9}$$

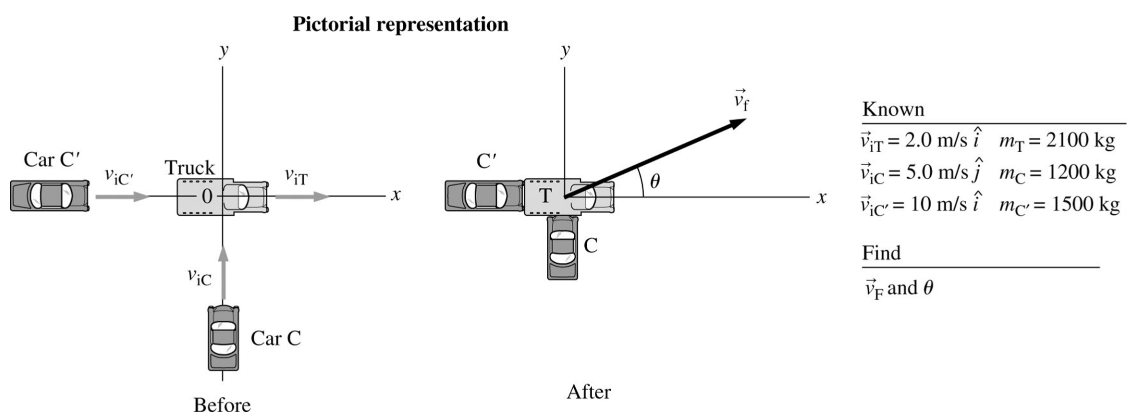
$$\Rightarrow (v_0)_m = \sqrt{18\ g\ L(1-\cos\theta)} = \sqrt{18(9.8\ \text{m/s}^2)(1.0\ \text{m})(1-\cos 50°)} = 7.9\ \text{m/s}$$

Problem 11-72

A 2100 kg truck is traveling east through an intersection at 2.0 m/s when it is hit simultaneously from the side and the rear. (Some people have all the luck!) One car is a 1200 kg compact traveling north at 5.0 m/s. The other car is a 1500 kg midsize traveling east at 10 m/s. The three vehicles become entangled and slide as one body. What are their speed and direction just after the collision?

11.72. Model: Model the truck (T) and the two cars (C and C′) as particles. The three forming our system stick together during their collision, which is perfectly inelastic. Since no significant external forces act on the system during the brief collision time, the momentum of the system is conserved.

Visualize:



Solve: The three momenta are

$$\vec{p}_{\text{iT}} = m_{\text{T}}\vec{v}_{\text{iT}} = (2100\text{ kg})(2.0\text{ m/s})\hat{i} = 4200\hat{i}\text{ kg m/s}$$

$$\vec{p}_{\text{iC}} = m_{\text{C}}\vec{v}_{\text{iC}} = (1200\text{ kg})(5.0\text{ m/s})\hat{j} = 6000\hat{j}\text{ kg m/s}$$

$$\vec{p}_{\text{iC}'} = m_{\text{C}'}\vec{v}_{\text{iC}'} = (1500\text{ kg})(10\text{ m/s})\hat{i} = 15{,}000\hat{i}\text{ kg m/s}$$

$$\vec{p}_{\text{f}} = \vec{p}_{\text{i}} = \vec{p}_{\text{iT}} + \vec{p}_{\text{iC}} + \vec{p}_{\text{iC}'} = (19{,}200\hat{i} + 6000\hat{j})\text{ kg m/s}$$

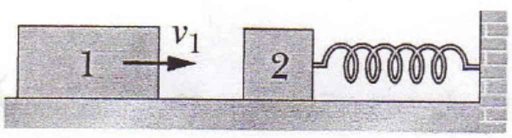
$$p_{\text{f}} = (m_{\text{T}} + m_{\text{C}} + m_{\text{C}'})v_{\text{f}} = \sqrt{(19{,}200\text{ kg m/s})^2 + (6000\text{ kg m/s})^2}$$

$$v_{\text{f}} = 4.2\text{ m/s},\ \theta = \tan^{-1}\frac{p_y}{p_x} = \tan^{-1}\frac{6000}{19{,}200} = 17^\circ\text{ above the }+x\text{-axis}$$

Assess: A speed of 4.2 m/s for the entangled three vehicles is reasonable since the individual speeds of the cars and the truck before entanglement were of the same order of magnitude.

Problem 11-A

In the figure below, block 2 (mass 1.0 kg) at rest on a frictionless surface and touching the end of an unstretched spring of spring constant 200 N/m. The other end of the spring is fixed to a wall. Block 1 (mass 2.0 kg), traveling at speed v_1 = 4.0 m/s, collides with block 2, and the two blocks stick together. When the blocks momentarily stop, by what distance is the spring compressed?

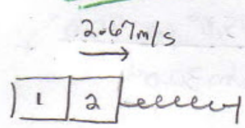


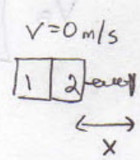
from conservation of momentum (right after collision):

$$P_f = P_i \longrightarrow (m_1 + m_2)\vec{V}_f = m_1\vec{V}_{1i}$$

$$\vec{V}_f = \frac{m_1\vec{V}_{1i}}{(m_1+m_2)} = \frac{(2.0\text{Kg})(4.0\text{m/s})}{(1.0\text{Kg} + 2.0\text{Kg})} \longrightarrow \vec{V}_f = 2.67\text{m/s}$$

now we can apply conservation of energy:





$$\tfrac{1}{2}(m_1+m_2)V_f^2 + \tfrac{1}{2}KX_f^2 = \tfrac{1}{2}(m_1+m_2)V_i^2 + \tfrac{1}{2}KX_i^2$$

($V_f = 0$, $X_i = 0$)

$$\tfrac{1}{2}Kx_f^2 = \tfrac{1}{2}(m_1+m_2)V_i^2 \longrightarrow X_f = \sqrt{\frac{(m_1+m_2)V_i^2}{K}} = \sqrt{\frac{(3.0\text{Kg})(2.67\text{m/s})^2}{200\text{N/m}}}$$

$$X_f = 0.33\text{m}$$